



# Measurements of total sea surface mean square slope field based on SWIM data

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# Outline

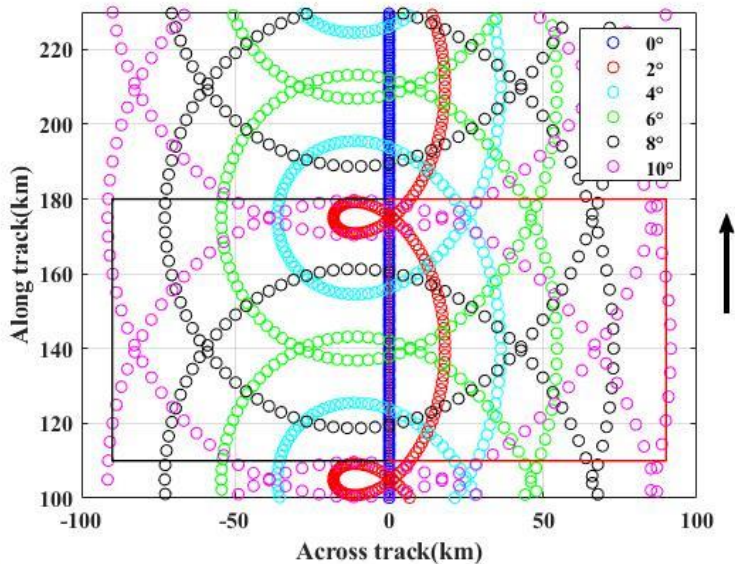
1. Motivation
2. Datasets
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# 1. Motivation

- ❑ Significant wave height (SWH) and mean square slope (mss) are two parameters that can describe sea-surface waves.
- ❑ PR/DPR probing at **one single azimuthal angle** can **yield a portion of the total mss**. If the dependence of the partial mss (called “ $mss_{xx}$ ” at the azimuthal angle) on azimuthal angles is derived, the total mss, which is irrelevant to ocean wave direction, can be calculated.
- ❑ CFOSAT/**SWIM** can measure NRCS profile from **various azimuthal angles**. Measurements at several azimuth angles are planned to be performed during an experiment on the Russian segment of the International Space Station in future.
- ❑ Using SWIM two-dimensional NRCS profiles, accurate and dependable total mss can be obtained by two methods.

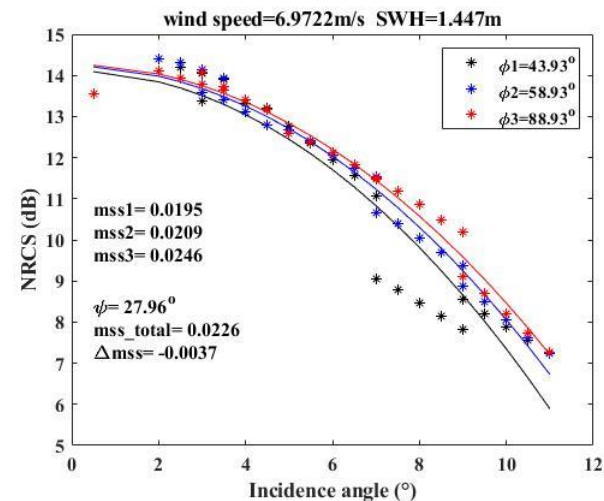
## 2. Datasets



Geometry of the footprints of the six beams of SWIM on the ocean surface. The black and red rectangles denote the box boundary

One L2 orbit comprises approximately  $500 \times 2$  boxes, including left and right boxes. In this case, the left and right boxes combine to form a **single large box**. When the land and ice flags are ignored, a total of 286658 large boxes are obtained during one year.

The profiles are labelled by “Sigma0\_mini\_profile” in L2, which are corrected for atmospheric attenuation,





## 3. Methodology

In previous well-known works, the authors used the assumption of **the isotropy of sea waves** with a Gaussian distribution function of heights to develop an algorithm to determine the slopes of the sea surface. The formula these authors used for the backscattering cross section has the following form:

$$\sigma_0(\theta) = \frac{|R_{eff}(0)|^2}{mss \cdot \cos^4 \theta} \exp\left[-\frac{\tan^2 \theta}{mss}\right] \quad (1)$$

**The mss here is the partial one** calculated by the NRCS profile at a single azimuthal angle.

### 3. Methodology

The  $mss_{xx}$  are calculated according to the NRCS profile in each azimuthal angle; in this case, the sea waves and wind are assumed to be uniform. However, the results show that the retrieved mss depends on the azimuthal angle; hence, this formula is not accurate for real sea waves.

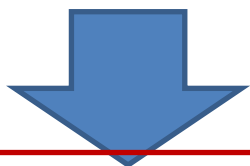
In the general case, the **anisotropy of the waves** is considered, and the formula for the backscattering cross section takes the following form

$$\sigma_0(\theta) = \frac{|R_{eff}(0)|^2}{2 \cos^4 \theta \sqrt{mss_{xx} mss_{yy} - mss_{xy}^2}} \times \exp \left[ - \frac{\tan^2 \theta}{2 (mss_{xx} mss_{yy} - mss_{xy}^2)} \cdot mss_{yy} \right] \quad (2)$$

When expressed in dB,  $\sigma_0$  is transformed as:

$$\sigma_0 (dB) + \frac{10}{\ln(10)} \ln(\cos^4 \theta) = A_0 + B_0 \tan^2 \theta \quad (3)$$

$$\sigma_0(\theta) = \frac{|R_{eff}(0)|^2}{2 \cos^4 \theta \sqrt{mss_{xx} mss_{yy} - mss_{xy}^2}} \times \exp \left[ -\frac{\tan^2 \theta}{2(mss_{xx} mss_{yy} - mss_{xy}^2)} \cdot mss_{yy} \right] \quad (2)$$



$$10 \log_{10} [\sigma_0(\theta)] + 10 \log_{10} (\cos^4 \theta) = 10 \log_{10} \left( \frac{|R_{eff}(0)|^2}{2 \cdot \sqrt{mss_{xx} mss_{yy} - mss_{xy}^2}} \right) - \frac{\tan^2 \theta \cdot mss_{yy}}{2(mss_{xx} mss_{yy} - mss_{xy}^2)} \cdot 10 \cdot \log_{10}(e)$$

(4)

$$A_0 = 10 \log_{10} \left( \frac{|R_{eff}(0)|^2}{2 \cdot \sqrt{mss_{xx} mss_{yy} - mss_{xy}^2}} \right) = \sigma_0 (dB)$$

$$B_0 = \frac{-mss_{yy}}{2(mss_{xx} mss_{yy} - mss_{xy}^2)} \cdot 10 \cdot \log_{10}(e)$$

$$B_0 \approx \frac{-1}{2mss_{xx}} \cdot 10 \cdot \log_{10}(e)$$



## 3. Methodology

$$mss_{total} = mss_{xx} + mss_{yy}$$

According to Karaev(2021), the dependence of  $mss_{xx}$  on the observation direction can be expressed as follows theoretically

$$mss_{xx}(\phi) = 0.5 \cdot mss_{total} + 0.5 \cdot \Delta mss \cdot \cos(2\phi - 2\phi_0)$$

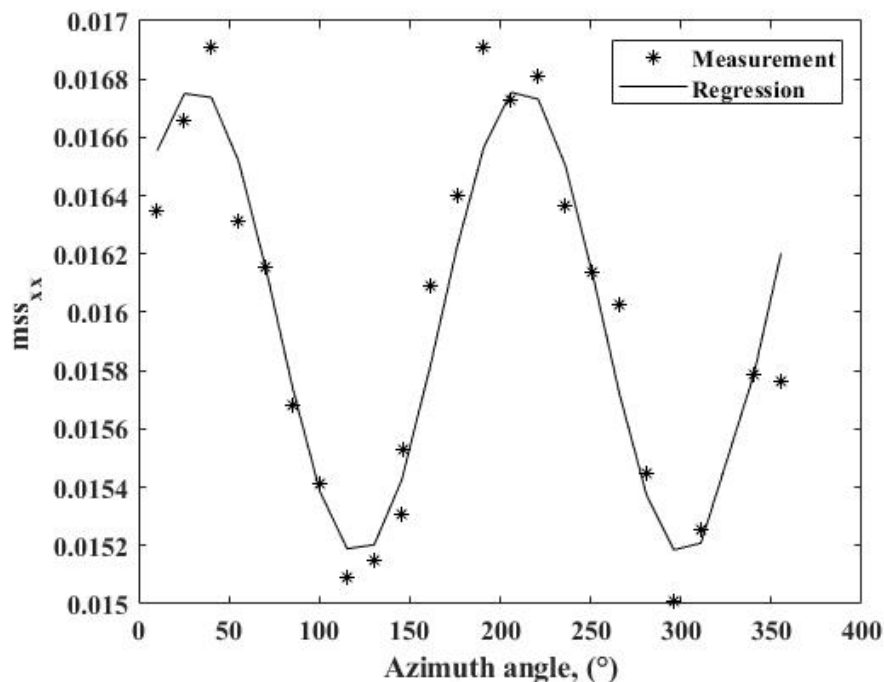
For SWIM, each large box can get 24 NRCS profiles, therefore, there are 24  $mss_{xx}$ , it is sufficient to calculate the  $mss_{total}$ .



## 3. Methodology

Two methods can be used to calculate the  $mss_{total}$ . **The first one** is curve fitting.

$$mss_{xx}(\phi) = 0.5 \cdot mss_{total} + 0.5 \cdot \Delta mss \cdot \cos(2\phi - 2\phi)$$



## 3. Methodology

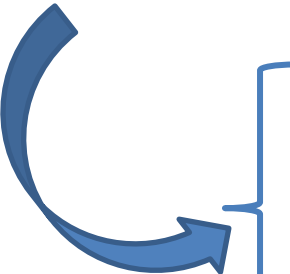
The **second method** is designed for the probing at several azimuth angles, according to Karaev(2021). In the general case of single-mode waves, measurement at three azimuthal angles is necessary to calculate the  $mss_{xx}$  at each azimuthal direction and find the total mss.

For a single-mode wave:

$$mss_{xx}(\phi_1) = 0.5mss_{total} + 0.5\Delta mss \cdot \cos(2\varphi - 2\phi_1)$$

$$mss_{xx}(\phi_2) = 0.5mss_{total} + 0.5\Delta mss \cdot \cos(2\varphi - 2\phi_2)$$

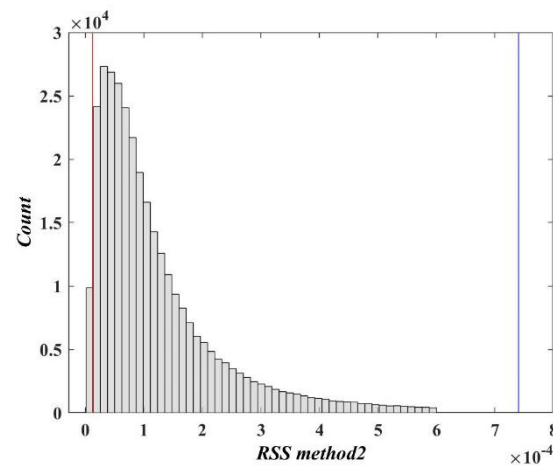
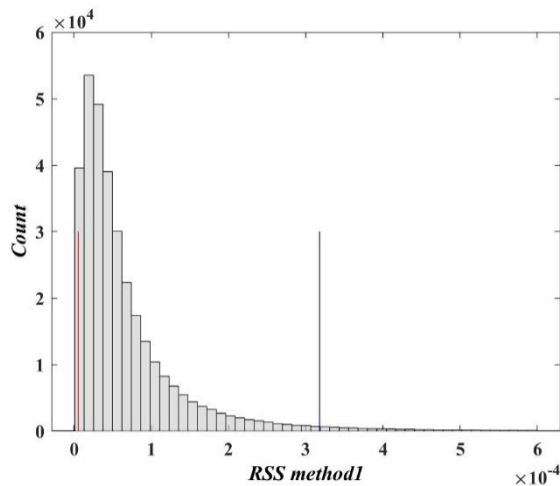
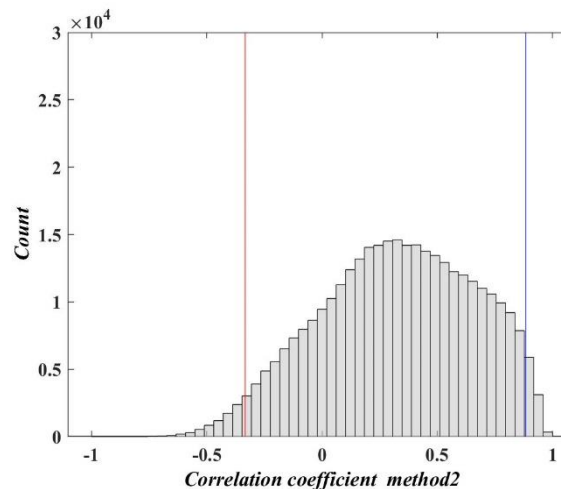
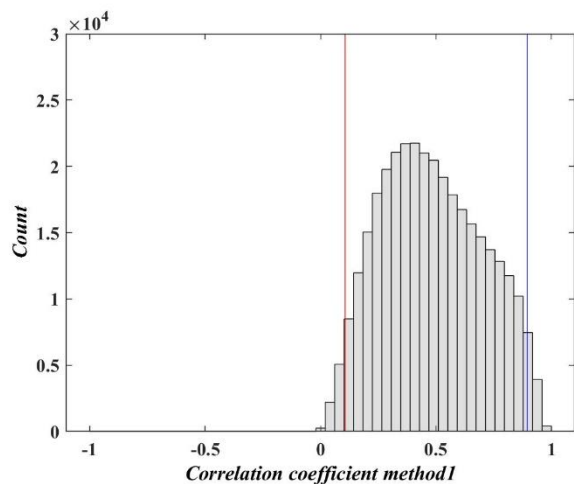
$$mss_{xx}(\phi_3) = 0.5mss_{total} + 0.5\Delta mss \cdot \cos(2\varphi - 2\phi_3)$$



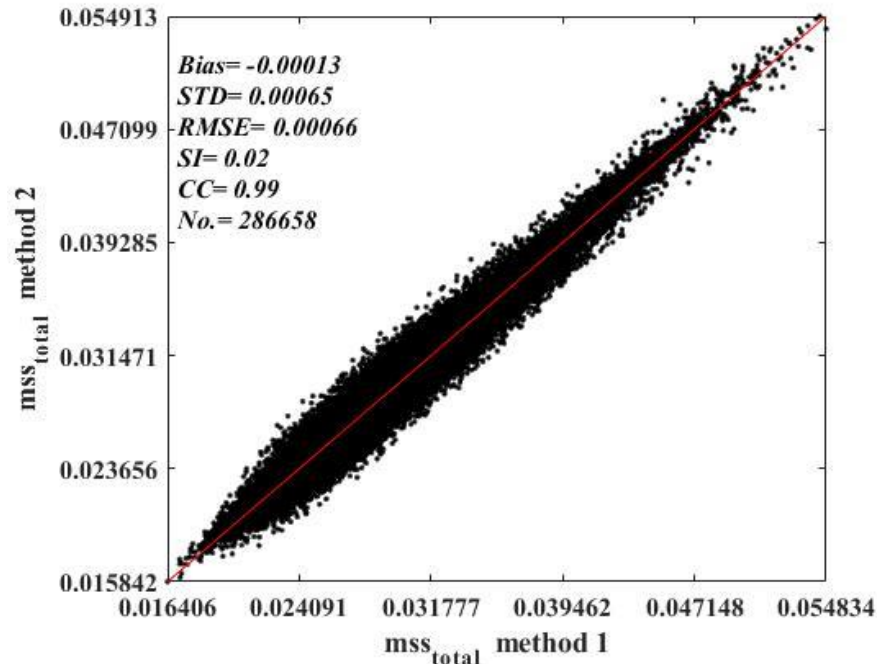
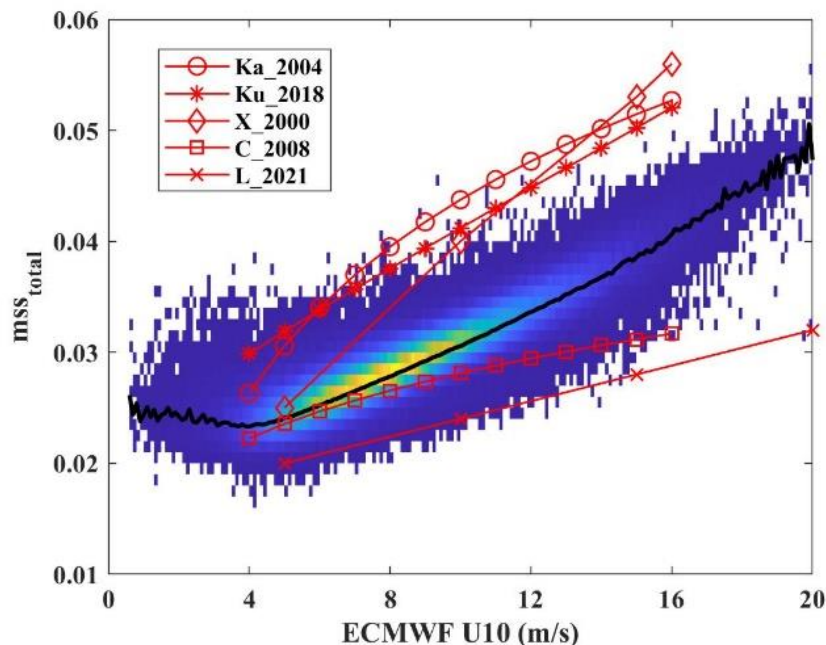
$$\varphi = 0.5 \times \arctan \left[ \frac{(B_2 - B_1) \sin(\phi_3 - \phi_1) \sin(\phi_1 + \phi_3) - (B_2 - B_1) \sin(\phi_3 - \phi_1) \cos(\phi_1 + \phi_3) - (B_3 - B_1) \sin(\phi_2 - \phi_1) \sin(\phi_1 + \phi_2)}{(B_3 - B_1) \sin(\phi_2 - \phi_1) \cos(\phi_1 + \phi_2)} \right]$$

$$mss_{total} = 2B_1 - \frac{2(B_2 - B_1) \cos(2\varphi - 2\phi_1)}{\cos(2\varphi - 2\phi_2) - \cos(2\varphi - 2\phi_1)}$$

# Quality control

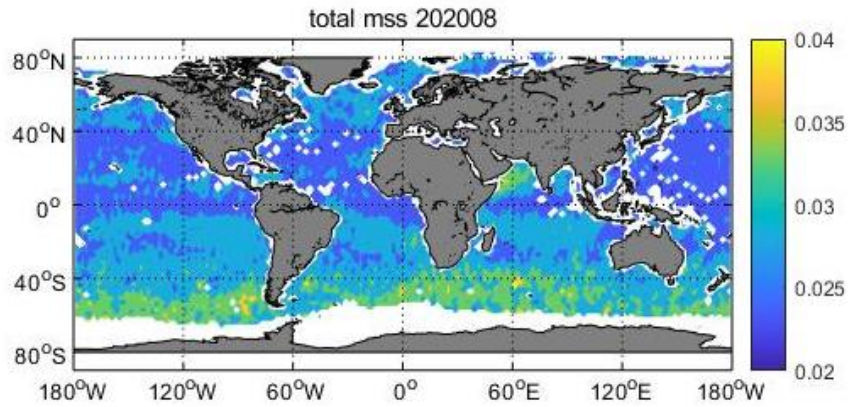


## 4 Results

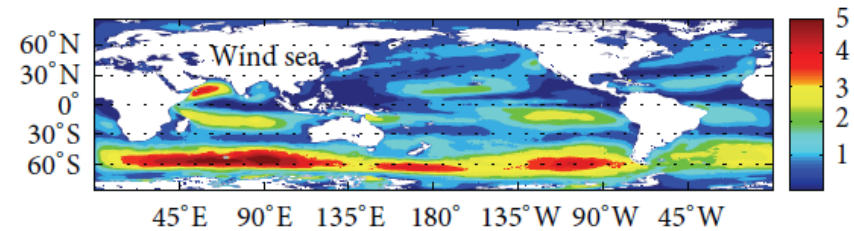


The black line shows the median of each wind speed bin. “Ka\_2004,” “Ku\_2018,” “X\_2000,” “C\_2008,” and “L\_2021” denote the omnidirectional mss acquired by means of Ka, Ku, X, C, and L band electromagnetic(EM) waves from Vandemark et. al., Chen et. al., Liu et. al., Hauser et. al. and Hwang et. al., respectively.

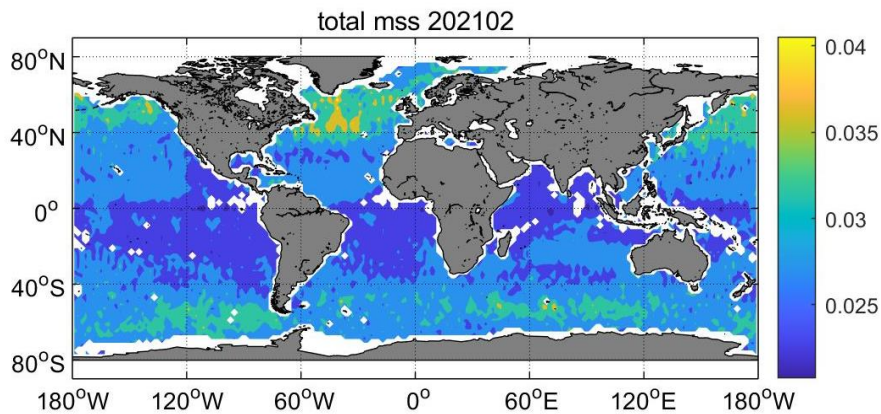
# 4 Results



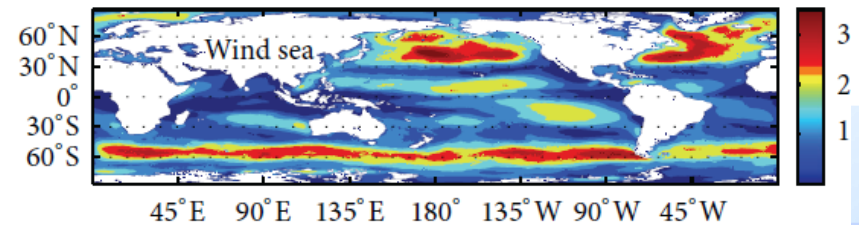
Jun.->Aug



K. Zheng, 2016,



Jan->Mar





## 5. Summary

The two methods are based on quasi-specular scattering theory to obtain the  $mss_{xx}$ , which is derived by the SWIM L2 product. They can determine the total mss of each big box of the SWIM product.

After quality control, the total mss from the two methods have the same general characteristics and nearly the same accuracy.

At wind speeds larger than 4 m/s, the total mss increases with increasing wind speed. At wind speeds lower than 4 m/s, this parameter slightly decreases with the wind speed, which may be attributed to the larger proportion of swell energy in this wind speed range than at other ranges.

The global distribution of the total mss is similar to the SWH distribution.

If a spaceborne radar can observe at three azimuth angles, the second method can provide the total mss.



Thank you for your attention!