Waves mean square slope (mss) estimation from CFOSAT/SWIM measurements

B. Gombert, J.F. Poustis, D. Hauser, P. Dubois, C. Tourain, A. Ollivier, M. Aversang

Outline

1. Definitions and disambiguation of mss

2. SWIM data used for mss inversion

3. mss estimation using Geometrical Optics (GO)

4. Second order models for mss inversion

The mean square slope (mss)

The mean square slope is an important ocean sea surface statistic. It is mainly governed by the **short wind waves** which are those supporting the wind stress,

It is known that mss increases with wind speed [*Cox & Munk*, 1954]

→ Strongly dependent on wind speed, slightly non-istropic

Following the Geometrical Optics (GO) theory, the mss can be analyzed from the variation of the radar cross-section (σ^0) with incidence angle (θ) at near nadir incidences.

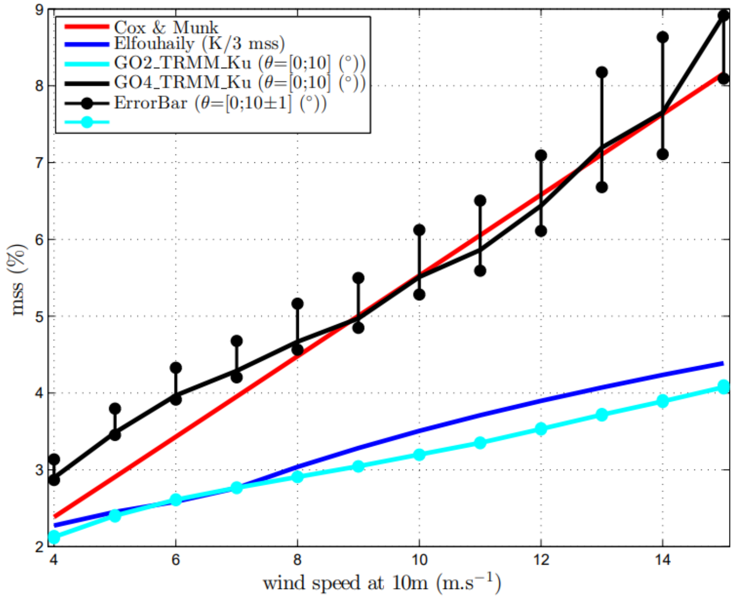
Several examples of mss estimation from radar data can be found on the literature

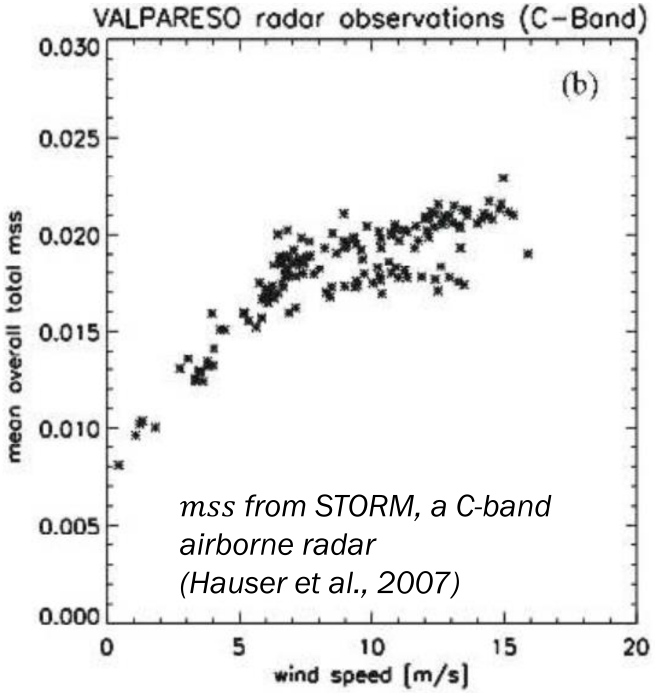
→ From Ku- and C-band airborne [*Jackson et al., 1992; Walsh et al., 1998; Hauser et al., 2007*]

→ From satellite observations in Ku- and Ka-band [Freilich, 2000, *Boisot et al., 2015; Nouguier et al., 2016*]

→ A recent study investigated the mss from CFOSAT/SWIM data [*Karaev et al., 2021*]

mss from TRMM, a Ku-band spaceborne radiometer (Boisot et al., 2015)





mss from STORM, a C-band airborne radar (Hauser et al., 2007)

Definitions and models from the literature

Definitions:

- Filtered mss from radar observations (GO model)

[Jackson et al. (1992); Chapron et al, 2000, Hauser et al. (2008); Freilich and Vanhoff (2003); Chu et al. (2012), Boisot et al. *(2015); F. Nouguier et al, 2016, Chen et al 2018, etc...)*

- › mss_{shape} : Only wavelength which are « seen by the radar" $(> \sim 3 * \lambda_{radar})$ contribute

- Total (unfiltered) mss: mss_T : all wavelengths are considered
  - › Optical measurements (Cox and Munk 1954),
  - › Radar measurements using higher order backscattering models

- Dependance on wind direction
  [Cox & Munk, 1954 , D. Hauser, 2007, Chu et al, 2012, Chen et al, 2018, etc]
  - › mss_{upwind} and $mss_{crosswind}$ are mss along and across the wind direction
  - › $mss_{total} = mss_{crosswind} + mss_{upwind}$
  - › $mss_{omni} = \frac{1}{2}\, mss_{totale}$ (also called mss_{iso})

Empirical models considered here:

- Cox & Munk [1954] : From optical observations of sun glitter

Clean-Sea : All wave wavelengths are observed (mss_T)

$$mss_{up,clean} = 0.00316\mathrm{U}_{10}$$
$$mss_{cx,clean} = 0.00192\mathrm{U}_{10} + 0.003$$
$$mss_{CM,clean} = mss_{up,clean} + mss_{up,clean}$$
$$= 0.00512\mathrm{U}_{10} + 0.003$$

Slick-Sea : Small waves are filtered out (equivalent to mss_{shape} with $* \ \lambda_{min} = 0.38\ m$)

$$mss_{up,slick} = 0.00078\mathrm{U}_{10} + 0.005$$
$$mss_{cx,slick} = 0.00084\mathrm{U}_{10} + 0.003$$
$$mss_{CM,slick} = 0.00156\mathrm{U}_{10} + 0.008$$

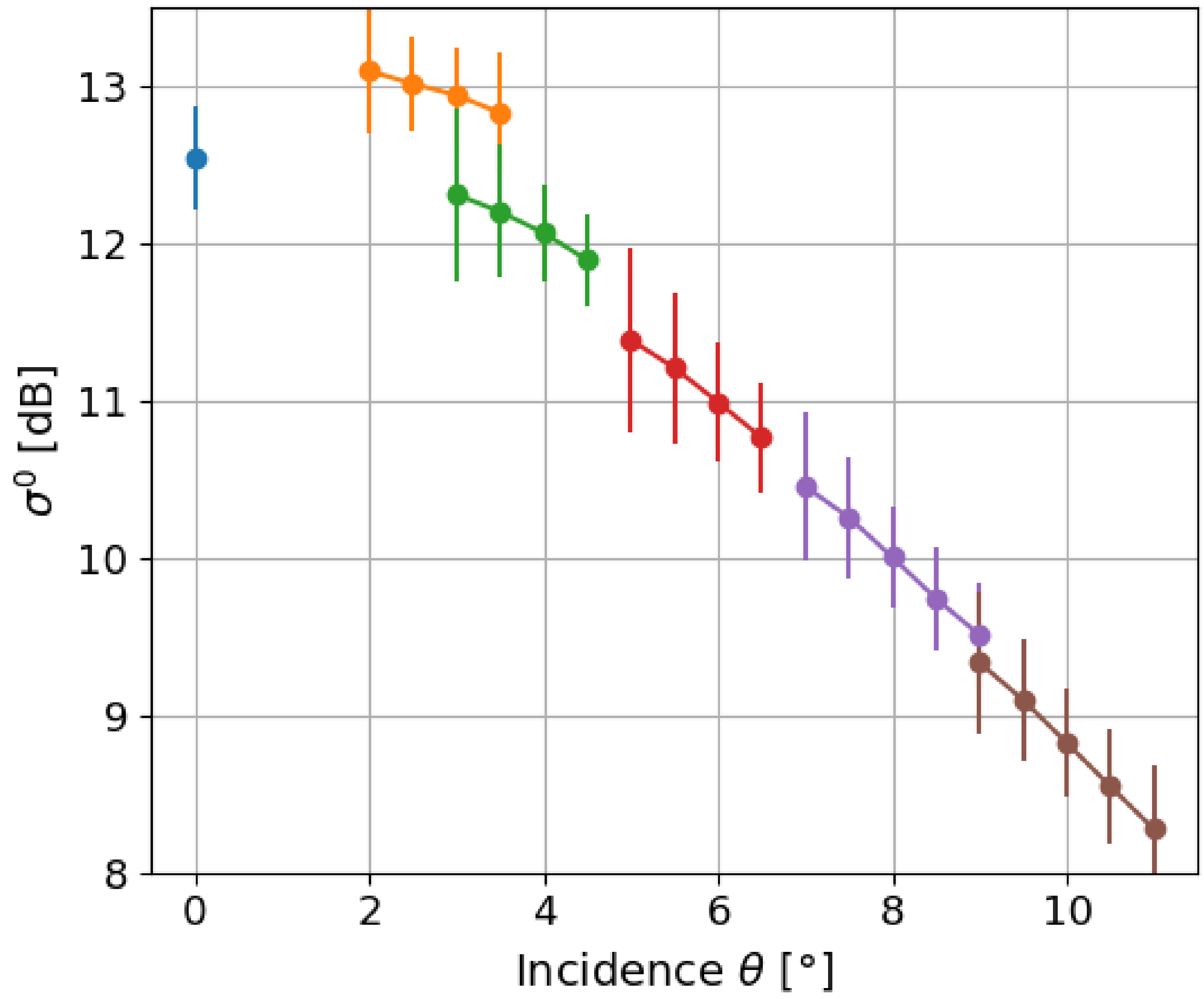
- Freilich & Vanhoff [2003] : from TRMM (Ku-band) observations (mss_{shape})

$$mss_{FV} = 0.0016\mathrm{U}_{10} + 0.016$$

SWIM data for mss estimation mss

Data used:

- SWIM L2 σ^0 « *mini-profiles* »,
  → 12 (one per azimuth) σ^0 mini-profiles per box
  → Nadir to ~11° incidence angles
  → Up to 9 σ^0 values per off-nadir beam
  → Beam 2° is not used in the inversion because of important inter-beam bias observed
- Data are from cycle 63 (15/01/2021 – 28/01/2021), with little to no anomalies due to microcuts
- Version V6.0 of AWWAIS processing
- Sea-ice and land data are filtered out



Analytical models – GO

Jackson [1981] : $\sigma^0(\theta)\cos^4(\theta) = |\mathcal{R}|^2\, p(\tan(\theta))$ with p the probability density of wave slopes

GO (or GO2) model: Assumes a Gaussian distribution of sea surface slopes

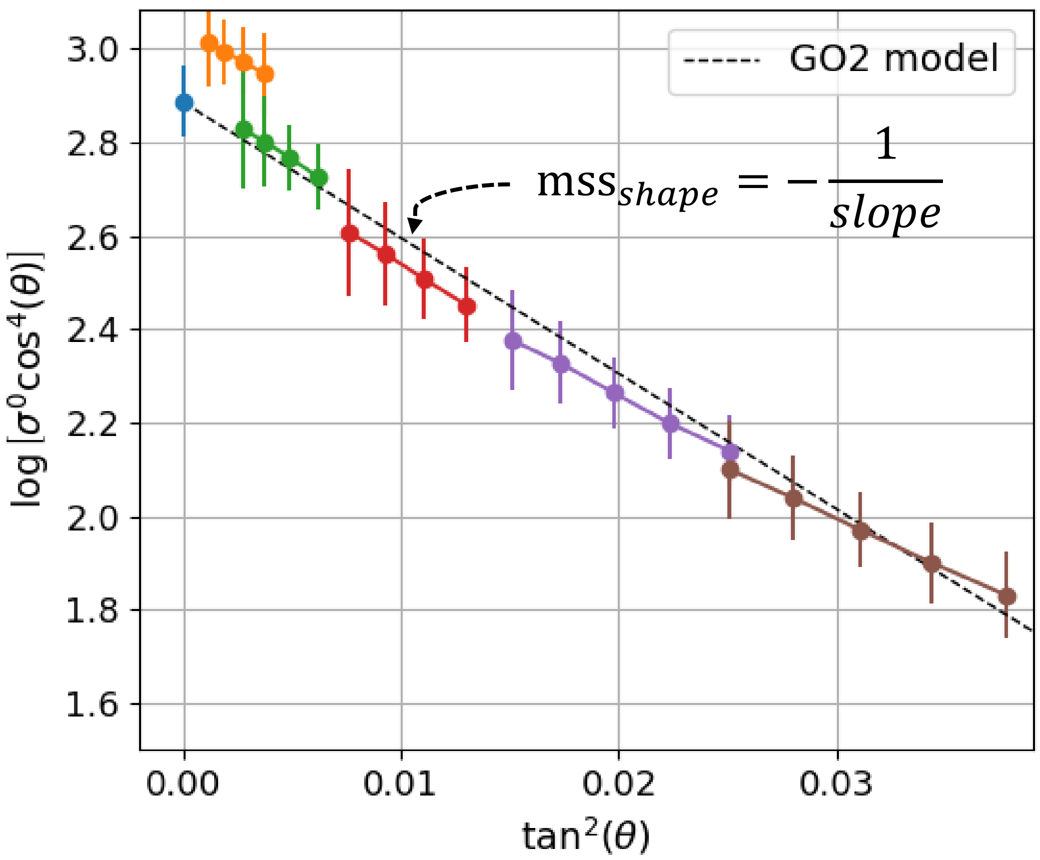
- Directional :

$$\sigma^0_{\mathrm{GO2,dir}}(\theta,\phi) = \frac{|\mathcal{R}|^2}{\sqrt{mss_{crosswind}\, mss_{upwind}}\cos^4\theta}\exp\left(-\frac{\tan^2\theta}{\mathbf{2}\ \mathrm{mss}_{shape}(\phi)}\right)$$

- Omnidirectional :

$$\sigma^0_{\mathrm{GO2,omni}}(\theta;\mathrm{mss}_{shape}) = \frac{|\mathcal{R}|^2}{\mathrm{mss_{shape}}\cos^4\theta}\exp\left(-\frac{\tan^2\theta}{\mathrm{mss}_{shape}}\right)$$

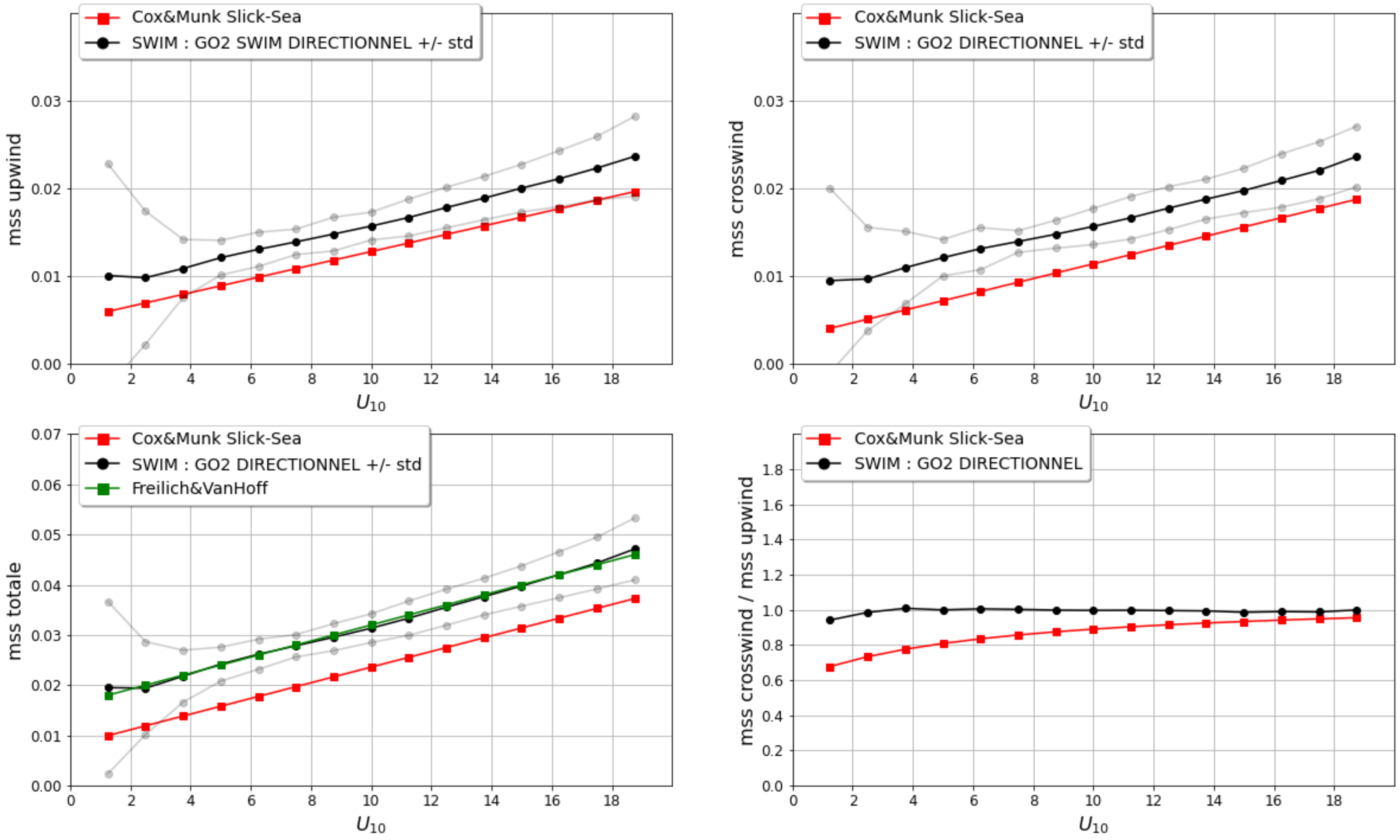
with, $\sigma_{omni}{}^0(\theta) = \frac{1}{2\pi}\int \sigma^0_{\mathrm{swim}}(\theta,\phi)\, d\phi$



mss_{shape} can simply be estimated with a linear regression of $\log(\sigma^0(\theta)\cos^4(\theta))$ vs $\tan^2(\theta)$

Analytical models – GO

- From SWIM $\sigma^0(\phi)$ measurements and co-located ECMWF wind, we compute mss_{shape} in up- and cross-wind directions, and total mss_{shape}
- Differences from Cox & Munk-slick can be explained by the different cutoff freq.
  → C&M $\lambda_{min} \approx$ 30 cm (see Wu et al, 1972)
  → Ku band $\lambda_{min} \approx$ 3-6
- There are no measurable difference between up- and cross-wind mss_{shape} measured with SWIM
- Remarkable agreement between total mss_{shape} from SWIM and Freilich & Vanhoff model (same λ_{min})



Analytical models – Second order models

The GO2 model provides the filtered mss (mss_{shape}) under a Gaussian assumption of the surface. In the literature, the approaches are used to obtain the **unfiltered** mss_T from near nadir radar observations:

- **Assume a Student distribution for probability density of wave slopes for wave** (*Guimbard, 2010*)
- **Use a higher order development of the geophysical approach (GO4,** *Boisot et al., 2015*)

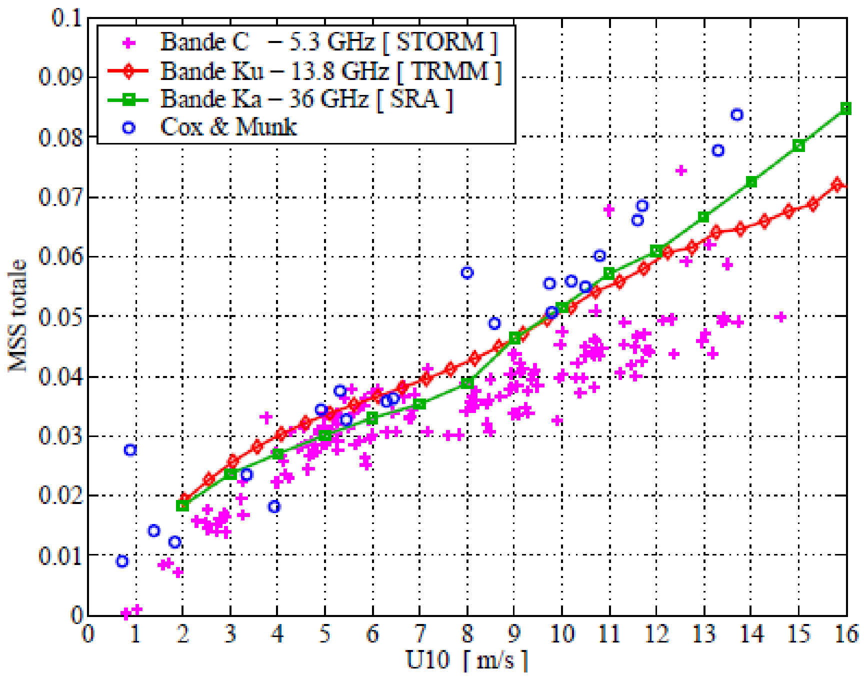
→ The Student law and G04 are two different approaches to the same concern, which could be expressed in a simple way as "accounting for the curvature effects". In principle when accounting for this effect, the mss inverted from radar near-nadir observations should correspond to the total (non-filtered) mss_T.

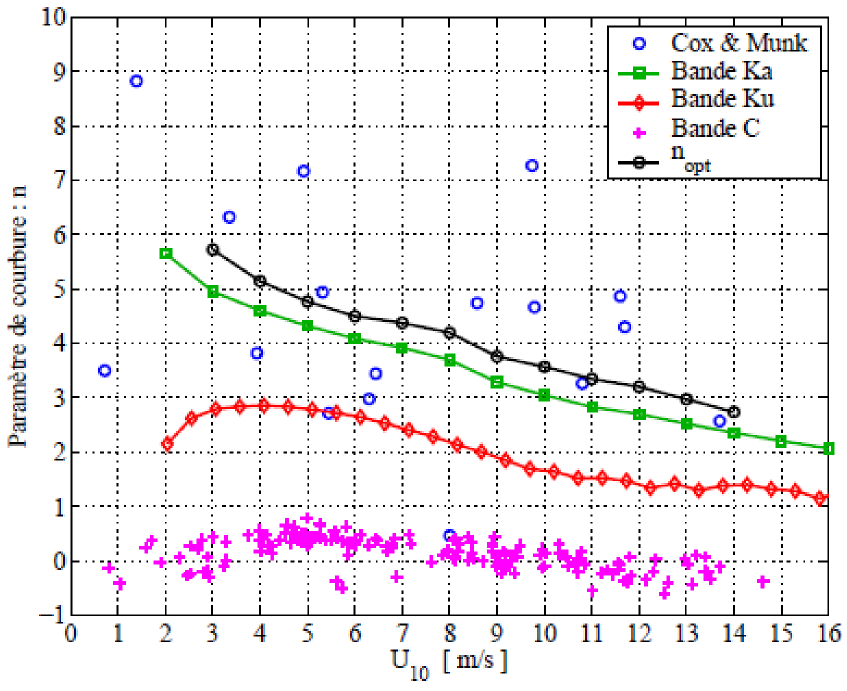
Analytical models – Second order models – Student Law

- **Assume a student distribution for probability density of wave slopes for wave** (*Guimbard, 2010*)

The goal here is to relax the assumption that wave slopes have a Gaussian PDF and use a Student distribution instead, which includes an additional curvature parameter ($n > 0$) and then uses the total mss instead the shape mss:

$$p(t = \tan^2(\theta)\ |n, \mathrm{mss}_T) = \frac{n+2}{\mathrm{mss}_T(n+1)}\left(1 + \frac{t}{\mathrm{mss}_T(n+1)}\right)^{-(n+3)}$$





Analytical models – Second order models – Student Law

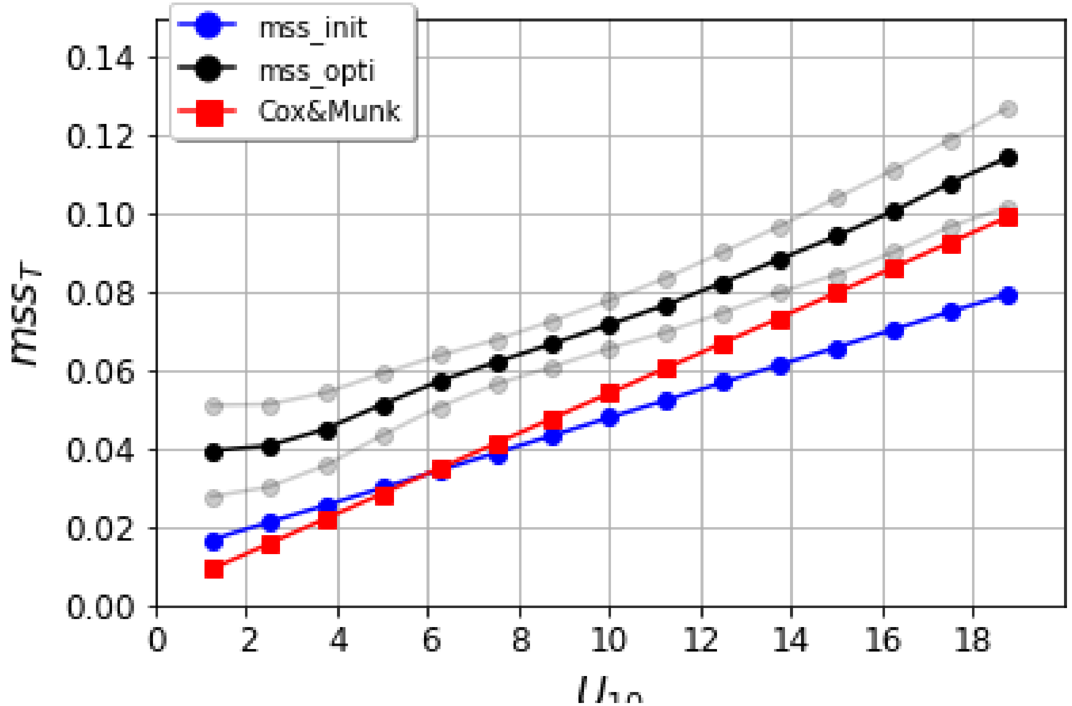
- **Student distribution for probability density of wave slopes for wave** (*Guimbard, 2010*)

› We search for the minimum of

$$J(|R(0)|^2, n, mss_T|\theta) = \frac{1}{2}|\sigma^0_{dB}(\theta) - \mathrm{A}(|R(0)|^2, \mathrm{mss_T}, \mathrm{n}) + 10(n+3)log_{10}\left(1 + \frac{\tan(\theta)^2}{mss_T\ (n+1)}\right)|^2$$

› We use a least square approach with the following constraints:

- Initialization
  - linear mss_T as a function of wind from *Guimbard (2010)*
  - n follows a normal law depending on wind speed from *Guimbard (2010)*
  - $|R(0)|^2 = 0.6$
- Constraints
  - $0 \leq mss_T \leq 0.2$
  - $0 \leq n \leq 3$
  - $0 \leq R_0 \leq 1$



Results:

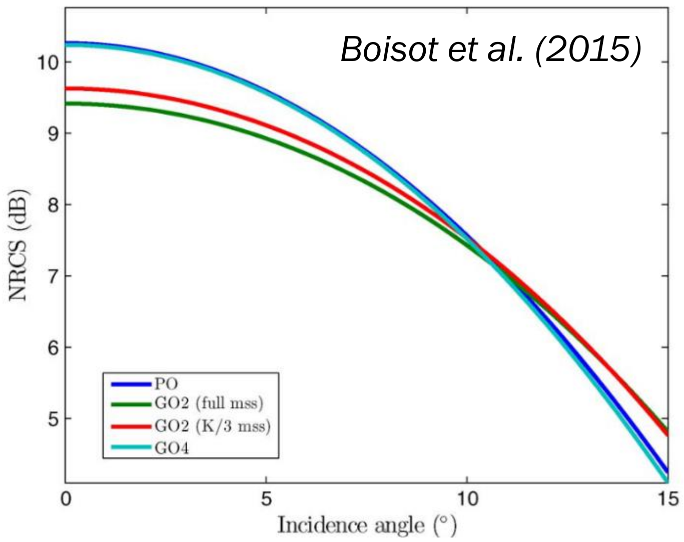
- mss_T has an expected behavior (linear increase with wind), but is overestimated compared to measurements from Cox & Munk
- n does not converge properly
- Inverted $|R(0)|^2$ values are overestimated

Analytical models – Second order models – GO4

- **GO4 model omnidirectional** (*Boisot et al., 2015*)

$$\sigma^0_{\mathrm{GO4,omni}}(\theta; mss_T) = \sigma^0_{\mathrm{GO2,omni}}(\theta; mss_T)\left[1+\frac{\alpha}{4}\left(\frac{\tan^4\theta}{\mathrm{mss}_\mathrm{T}^2} - 4\frac{\tan^2\theta}{\mathrm{mss}_\mathrm{T}} + 2\right)\right]$$

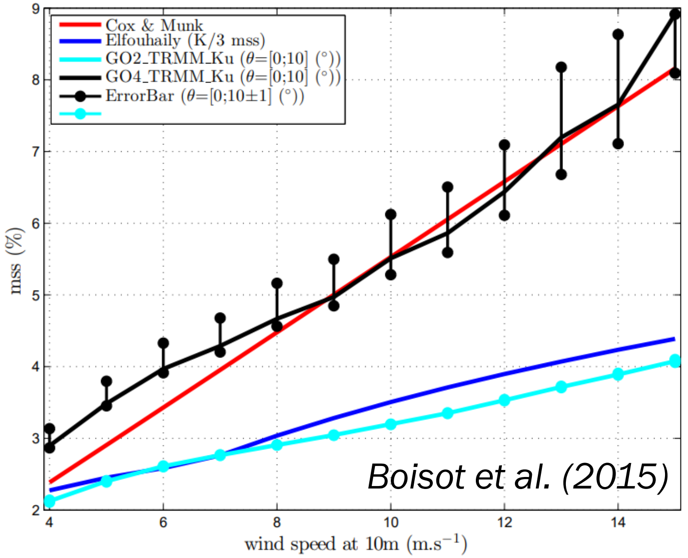
with $\alpha = \frac{msc}{Q_z^2 mss_T^2}$ (+kurtosis param. assumed to be 0)



→Better description of near-nadir microwave scattering from the sea-surface, with an accuracy comparable to the PO model.

→Adds an effective "mean square curvature" msc (but ill-defined)

→It uses the omnidirectional σ^0_{omni}. The directional mss is outside the scope of this study



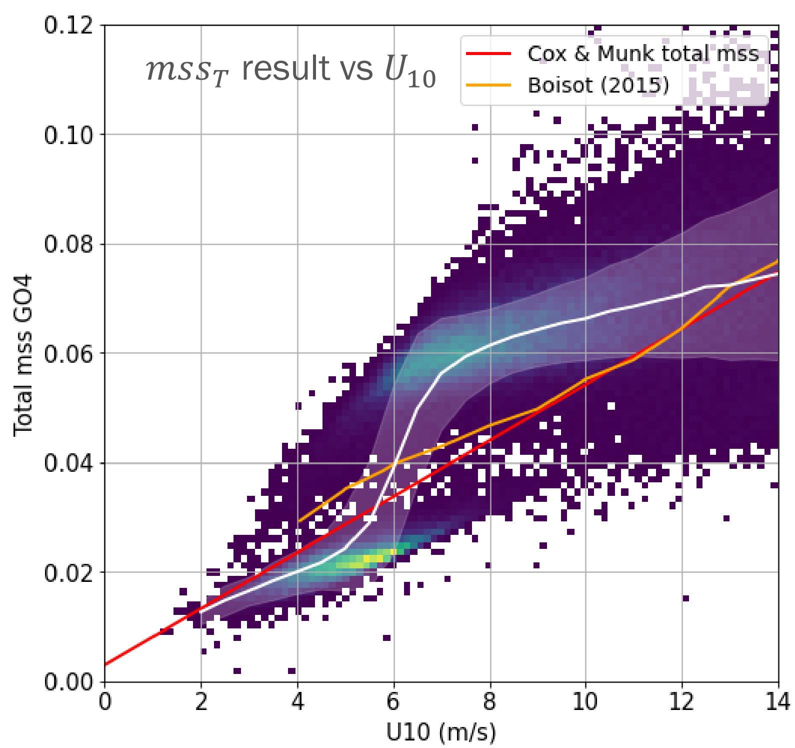
Analytical models – Second order models – GO4

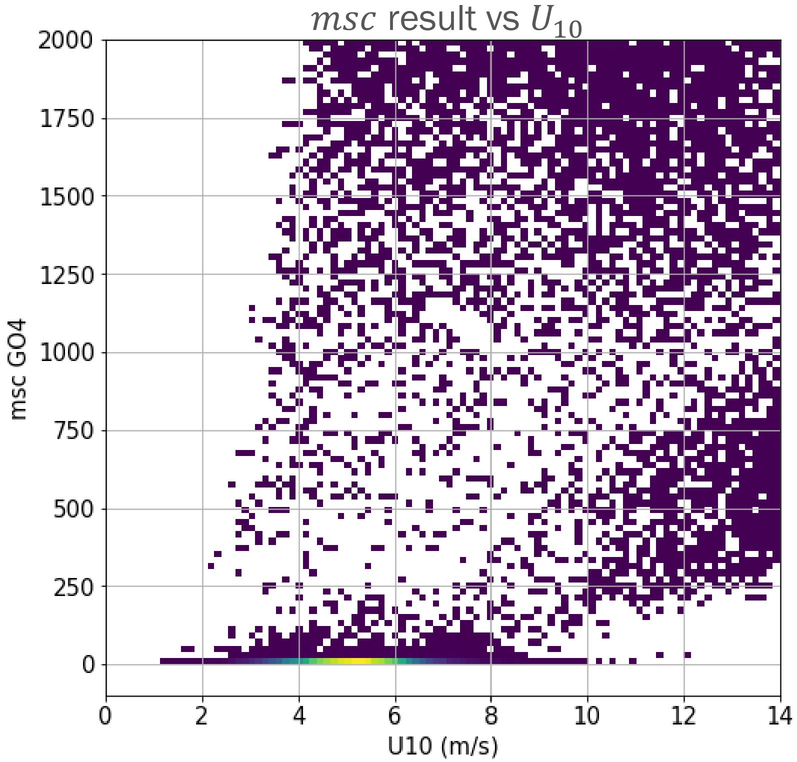
Inversion approach #1

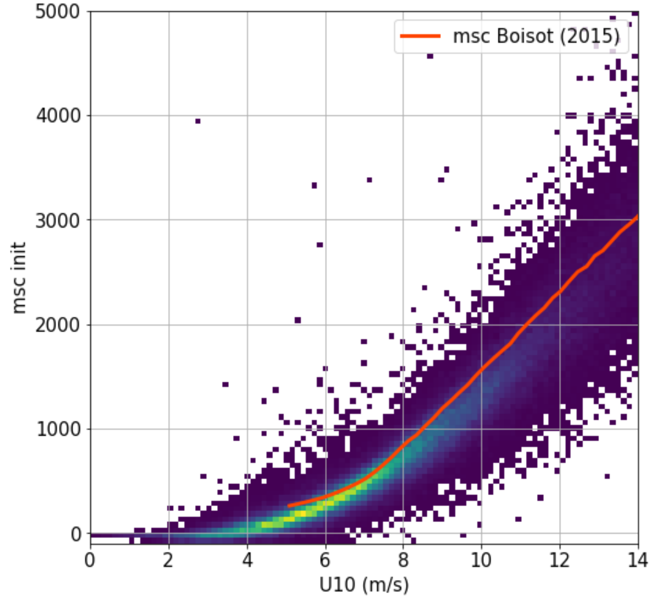
→ Estimation is made by minimizing $\sigma^0_{omni} - \sigma^0_{\mathrm{GO4}}(\theta, mss, msc)$ with a least-square approach

→ As a first-guess we use mss_{CM} and $msc_{init} = 8\frac{2\pi}{\lambda} mss^2_{CM}\left(\frac{\sigma^0_{nadir} mss_{CM}}{R^2} - 1\right)$

→ Inversion of two parameters : mss_T and msc







➔ Poorly converging inversion
➔ msc constrained to lower bound
➔ Two regimes of mss_T, not clearly understood

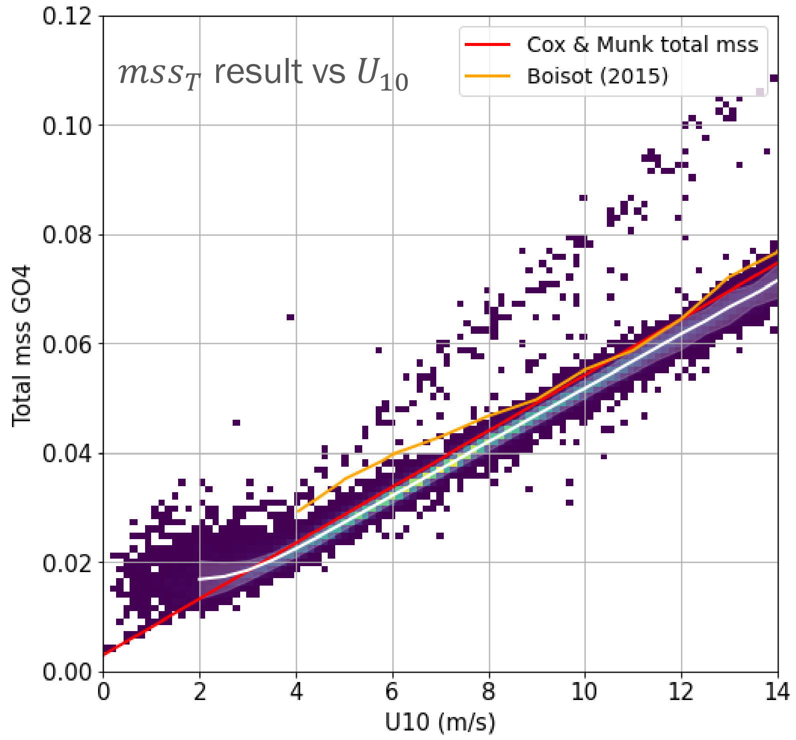
Analytical models – Second order models – GO4

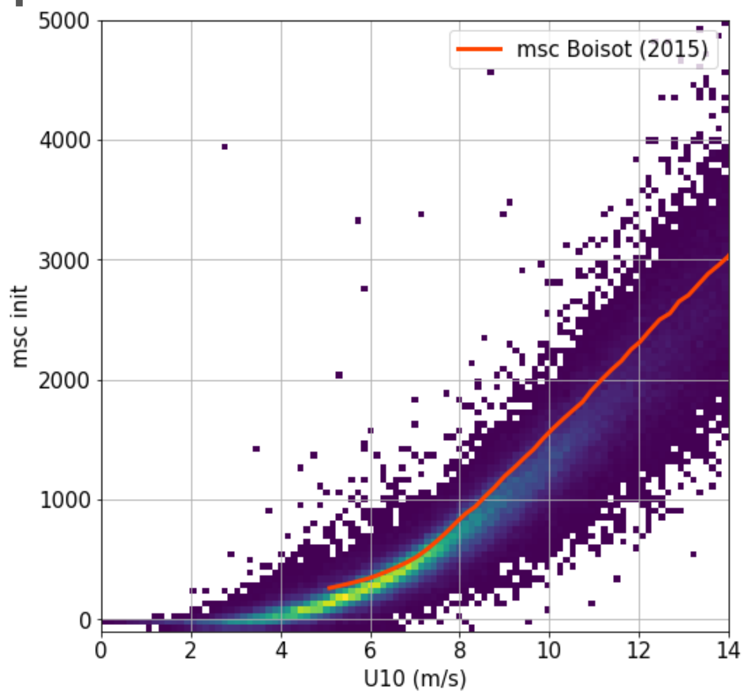
Inversion approach #2

→ Estimation is made by minimizing $\sigma^0_{omni} - \sigma^0_{\mathrm{GO4}}(\theta, mss, msc)$ with a least-square approach

→ As a first-guess we use mss_{CM} and $msc_{init} = 8\frac{2\pi}{\lambda} mss^2_{CM}\left(\frac{\sigma^0_{nadir} mss_{CM}}{R^2} - 1\right)$

→ Inversion of <u>one</u> parameters : mss_T . msc is fixed to msc_{init}





→ Not ideal, as msc depends on mss_{CM} …

→ … but significantly improved mss_T estimation

→ Above 3 m/s wind speeds, remarkable agreements between TRMM estimates (Boisot et al., 2015) and Cox & Munk empirical model

Conclusions

We demonstrated the capability to estimate, to some degrees, different values of mss with SWIM L2 σ^0 data

mss_{shape} (filtered mss)

- We find a solid estimation of total and directional mss_{shape} using the GO2 model and a linear regression.
- The inferred values are very consistent with the literature

mss_T (unfiltered mss)

- Student distribution for wave slopes PDF is a promising lead, that needs to be investigated further. At this stage, inversion offers reasonable mss values. There seem to be a bias compared to Cox & Munk measurements, but the dependance on wind speed is comparable.

- Using the GO4 model gives very good mss_T results, but with a strong assumption on msc which is fixed in the inversion. The inferred mss_T strongly agrees with TRMM values from *Boisot et al., (2015)* and optical measurements from *Cox & Munk (1953)*

→ For these finer models and complete inversions of all parameters (mss, msc, n), we are likely reaching the current limit of σ^0 mini-profile accuracy.

Solutions include either a selection of "high-quality" mini-profiles (with TBD criterions) and/or improvements in radiometric calibration

➔ **Possibility to includes $\mathbf{mss_{shape}}$ and/or $\mathbf{mss_T}$ to SWIM L2 products**

BACKUP

Student law vs GO4

The GO4 model uses a 4th order development of the correlation (instead of 2nd order for GO2)

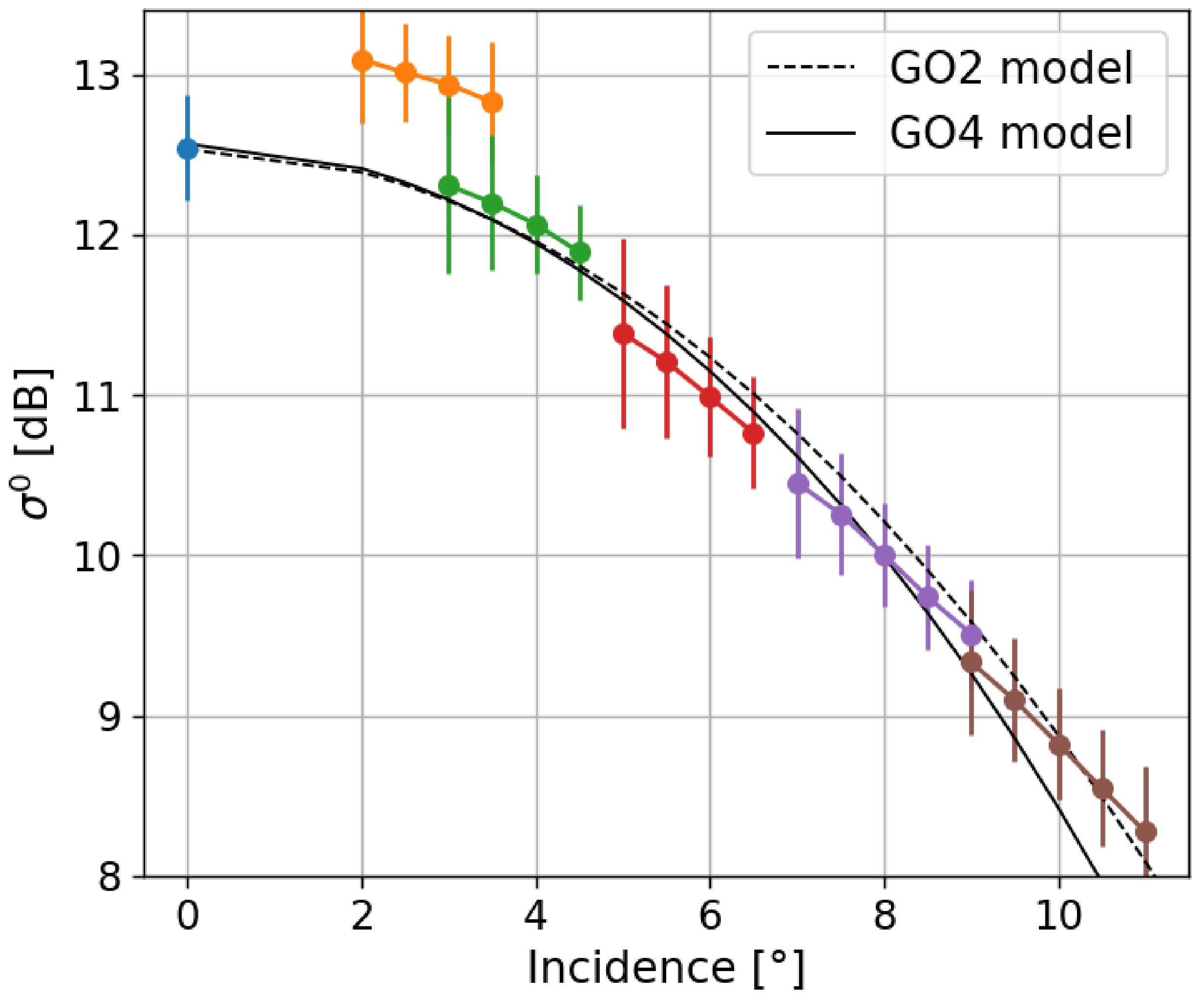
GO2: $\rho(r) = \rho(0) - \frac{\mathrm{mss}_{shape}}{2} r^2 + \mathrm{o}(r^2)$

GO4: $\rho(r) = \rho(0) - \frac{\mathrm{mss}_T}{2} r^2 + \frac{\mathrm{msc}}{32} r^4 + \mathrm{o}(r^4)$, msc is the spectral moment of order 4, related to the curvature.

(Guimbard, 2010) identified the Kurtosis of the student law, $\kappa_4 = \frac{2\mathrm{mss}_T^2}{\mathrm{n}}$, to the term $\frac{\mathrm{msc}}{(2\mathrm{K}\cos\theta)^2}$, arising from the use of the 4th order correlation while solving the Kirchhoff Integral ($\propto \sigma^0$).

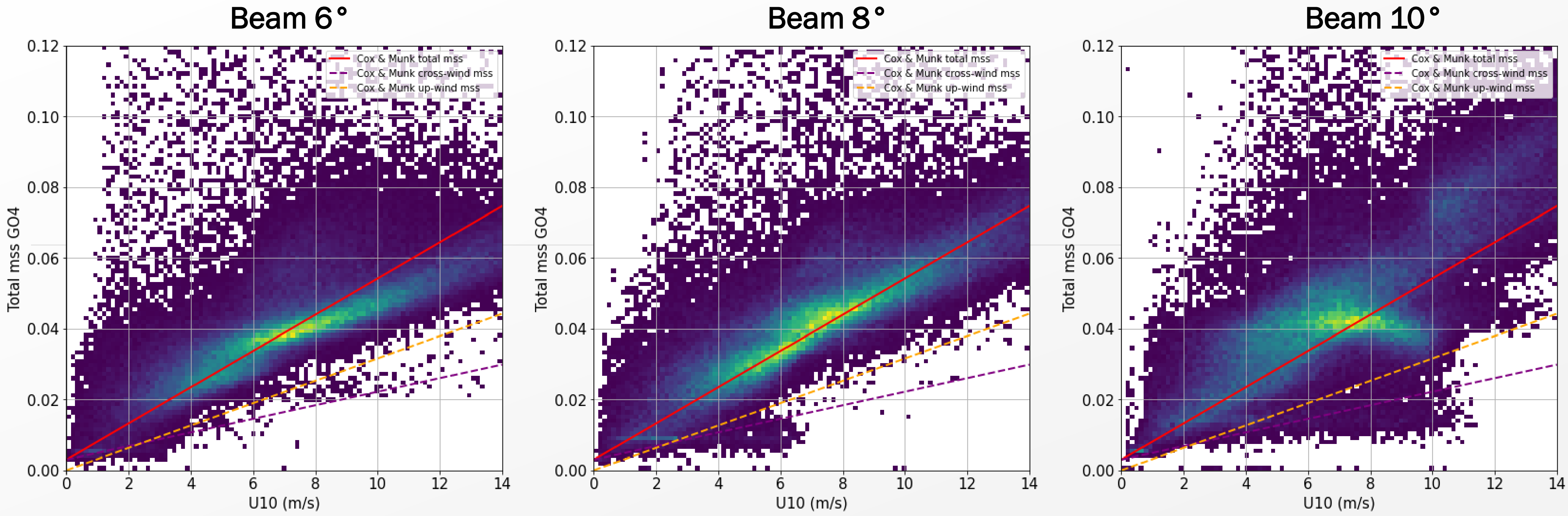
The GO4 parameter $\alpha = \frac{\mathrm{msc}}{(2K\cos\theta)^2 \mathrm{mss}_T^2}$ and the parameter n of the Student law are then straightforwardly related.

Note: Using the 2nd order expansion of $\rho(r)$ and solving Kirchhoff Integral is strictly equivalent to choosing a Gaussian distribution for the probability density of wave slopes (*Jackson, 1981:* Physics Optical (PO) model). Therefore, the terminology GO(2) is often used, by abuse of language, to refer to the PO model under Gaussian assumption.


GO2 model
GO4 model
σ^0 [dB]
Incidence [°]
13
12
11
10
9
8
0
2
4
6
8
10

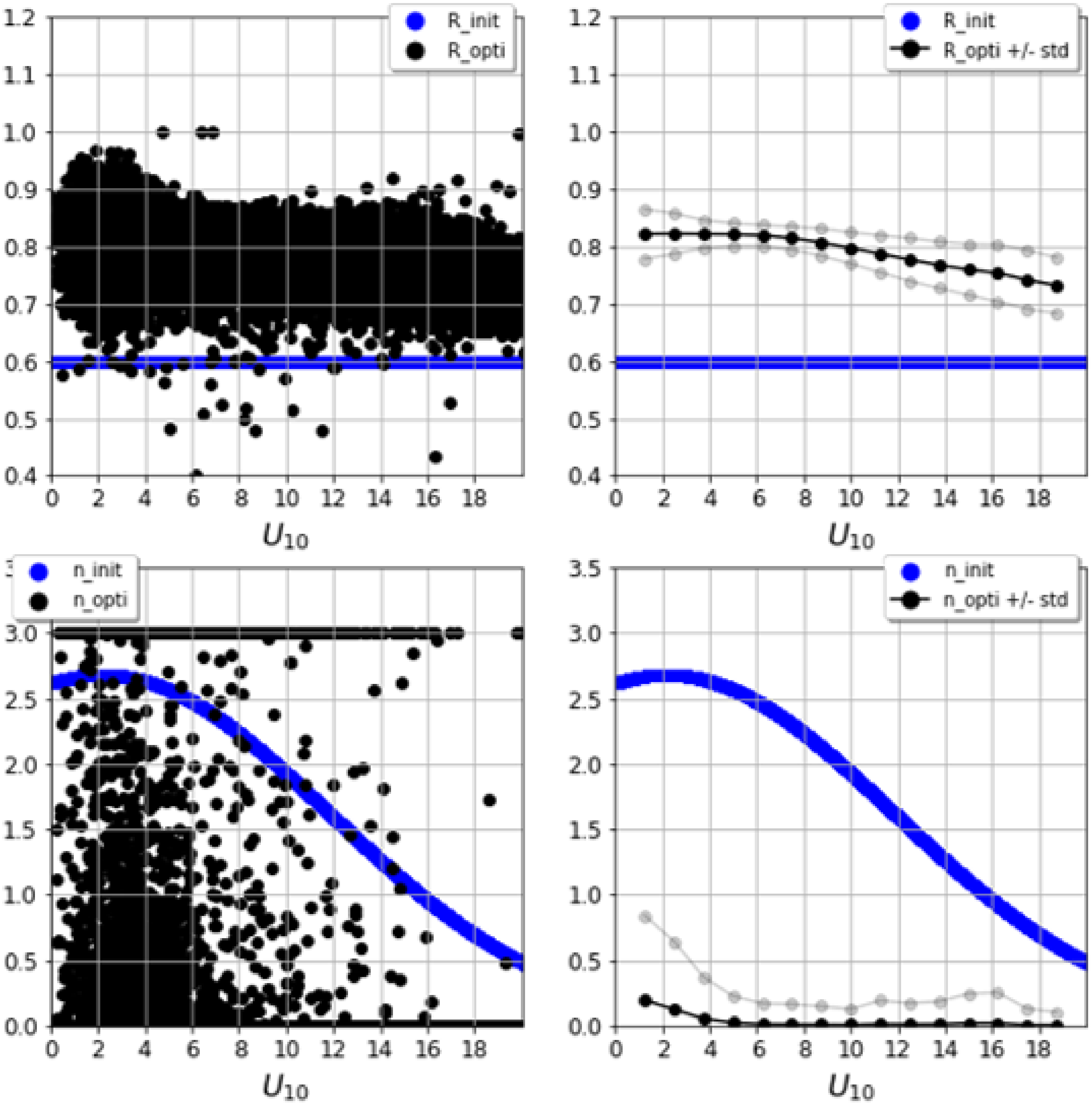
Calcul de mss à partir de données σ^0 SWIM

Calcul de $\boldsymbol{mss_T}$ isotrope à partir de données $\boldsymbol{\sigma^0_{\text{swim,iso}}(\theta)}$ et modèle GO4



Cox & Munk

CLS
COLLECTE LOCALISATION SATELLITES



R_init
R_opti
R_init
R_opti +/- std
U_{10}
U_{10}
n_init
n_opti
n_init
n_opti +/- std
U_{10}
U_{10}